Announcement: Midterm 1 moved to Mon, Jan 31 (still in-person)

Today §2.4: Limits and Continuity

+ finish §2.3: Basic Limit Laws

Poll If you're interested in in-person outdoor office hours next week, which time is best?

A) Mon, 11am B) Mon, 1pm C) Wed, 9am D) Wed, 1pm

In-person office hour, Mon, Jan 10, 1-2pm at Art of Espresso

Basic Limit Laws

If $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ exist, then:

1) $\lim_{x\to c}\left[f(x)+g(x)\right] = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$ (Sum Law)

2) For any constant k:
$\lim_{x\to c} k\cdot f(x) = k \cdot \lim_{x\to c} f(x)$ (Const. mult. Law)

3) $\lim_{x\to c} f(x)g(x) = \lim_{x\to c} f(x) \cdot \lim_{x\to c} g(x)$ (Product Law)

4) $\lim_{x\to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$, assuming $\lim_{x\to c} g(x) \neq 0$ (Quotient Law)

5) For any constant r, $\leftarrow 1/2$
$\lim_{x\to c}\left[f(x)\right]^r = \left(\underbrace{\lim_{x\to c} f(x)}_{-1}\right)^r$, assuming $\left(\lim_{x\to c} f(x)\right)^r$ is defined (Powers & Roots Law)

Note book assumes $r = P/q$ is rational, but this works for any real r: $2, \frac{1}{2}, -1, \pi$

<u>Ex</u> (2a) $\displaystyle\lim_{x\to -1} \frac{x+6}{2x^4} = \frac{(-1)+6}{2(-1)^4} = \boxed{\frac{5}{2}}$

Uses $\displaystyle\lim_{x\to c} k = k$ & $\displaystyle\lim_{x\to c} x = c$

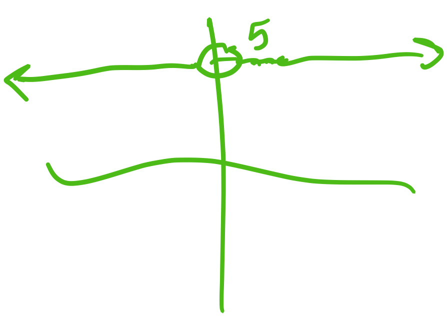
and sum law, product law, quotient law

<u>Q</u> Does the assumption "$\displaystyle\lim_{x\to c} f(x)$ exists" allow for $\displaystyle\lim_{x\to c} f(x) = \infty$?

<u>A</u> No! Must be a <u>finite</u> <u>real</u> <u>number</u>

<u>Ex</u> (≈ 3) $f(x) = 5x^2$, $g(x) = x^{-2}$

$$\lim_{x\to 0} f(x)g(x) \overset{?}{=} \lim_{x\to 0} f(x) \lim_{x\to 0} g(x) \quad \text{(Prod. Law?)}$$



$$= \lim_{x\to 0} 5x^2 \cdot \lim_{x\to 0} x^{-2}$$

$$= 0 \cdot \infty = ?? \text{ BAD}$$

More on this in §2.5

The product rule does <u>not</u> work here

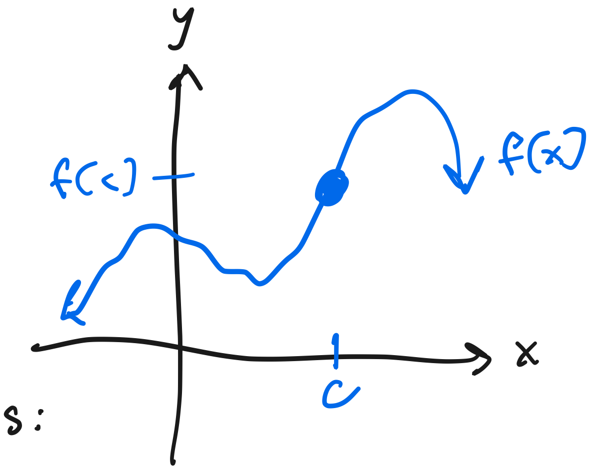
<u>Instead</u> $\displaystyle\lim_{x\to 0} \underbrace{5\cancel{x^2 \cdot x^{-2}}}_{} = 5$ $\quad x^2 \cdot x^{-2} = 1$

$= 5$ <u>except</u> when $x = 0$

§2.4 Limits & Continuity

<u>Def</u> A function f is <u>continuous</u> at $x = c$ if $\displaystyle\lim_{x\to c} f(x) = f(c)$. (Otherwise we say

f is discontinuous at c).

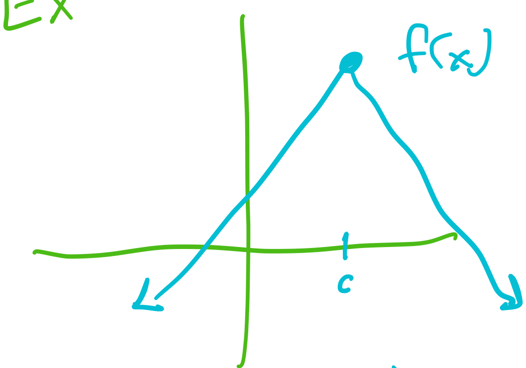


Geometric meaning: the graph of f has no holes, jumps, or gaps

Notice For f to be continous at $x = c$, need 3 conditions:

1) $f(c)$ is defined
2) $\lim_{x \to c} f(x)$ exists
3) $\lim_{x \to c} f(x) = f(c)$

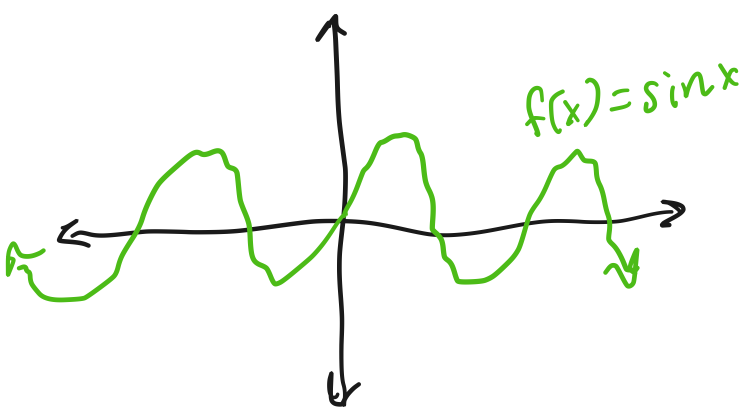
Ex

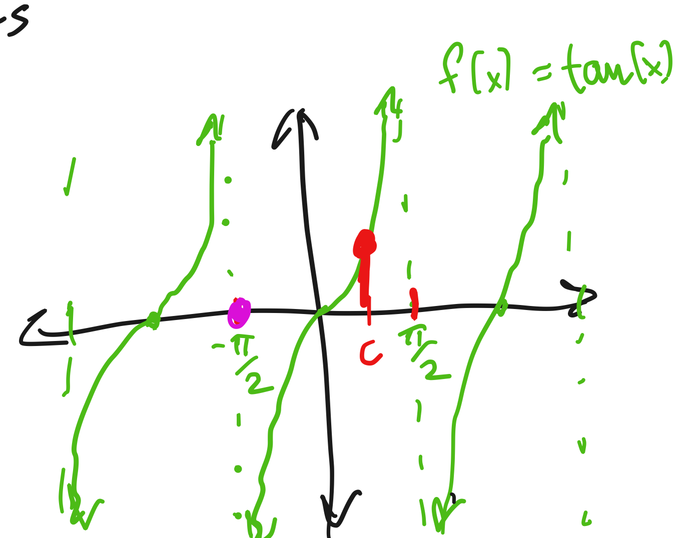


This function is cont. at $x = c$

Examples

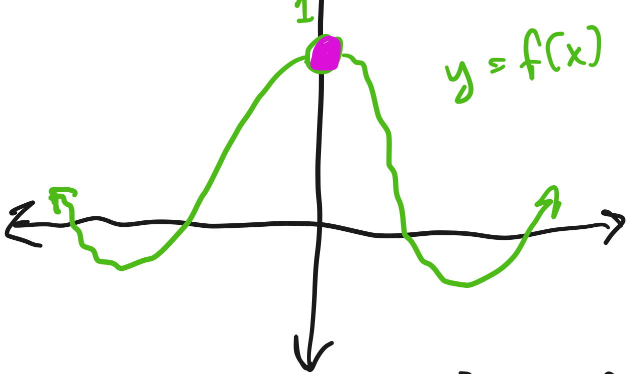
- Polynomials, exponentials, $\sin(x)$, $\cos(x)$ are continuous at all $x \in \mathbb{R}$ ← real numbers
  ↑ "in" "element of"
- $\sqrt{x}$, x^n, $\ln(x)$, $\tan(x)$ continuous at all $x = c$ in their domains





Examples/types of discontinuities:

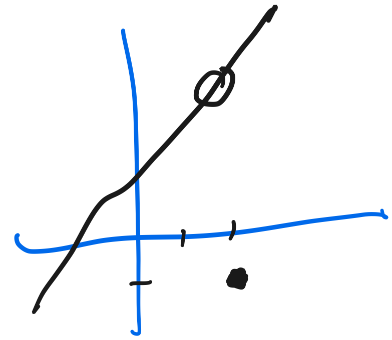
Ex $f(x) = \frac{\sin(x)}{x}$ at $x = 0$ (Removable discontinuity)



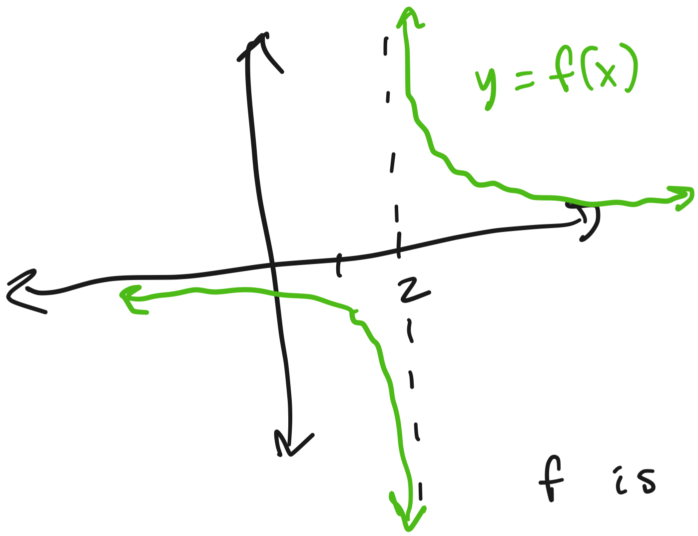
Not continuous at $x=0$ since $f(0)$ is undefined

$$f(x) = \begin{cases} x+1 & \text{if } x=2 \\ -1 & \text{if } x=2 \end{cases}$$

$$g(x) = \begin{cases} \frac{\sin(x)}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$



Ex $f(x) = \frac{1}{x-2}$ at $x=2$ (Infinite discontinuity)

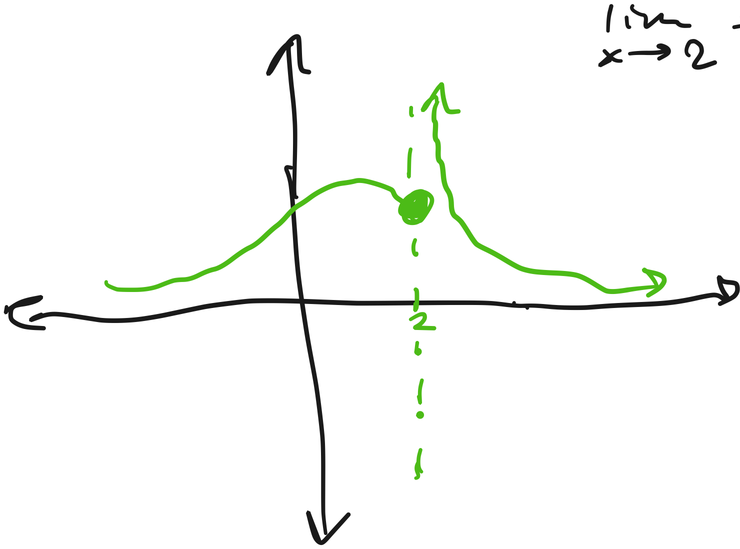


$\lim_{x\to 2^+} f(x) = +\infty$

$\lim_{x\to 2^-} f(x) = -\infty$

$f(2) =$ undefined

f is not cont. at $x=2$ since $\lim_{x\to 2} f(x) \neq f(2)$



- "left-continuous" at $x=2$
- infinite discontinuity at $x=2$